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Relating the Komargodski-Seiberg and Akulov-Volkov actions: Exact nonlinear field redefinition

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Abstract

This paper constructs an exact field redefinition that maps the Akulov-Volkov action to that recently studied by Komargodski and Seiberg in arXiv:0907.2441. It is also shown that the approach advocated in arXiv:1003.4143v2 and arXiv:1009.2166 for deriving such a relationship is inconsistent.

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1 Introduction

The Akulov-Volkov (AV) action [1] is the second oldest supersymmetric theory in four space-time dimensions. It describes the low-energy dynamics of a massless Nambu-Goldstone fermionic particle which is associated with the spontaneous breaking of rigid supersymmetry and is called the Goldstino (see [2] for a nice review of the AV model and related concepts). According to the general theory of the nonlinear realization of $\mathcal{N} = 1$ supersymmetry [1, 3, 4, 5, 6], the AV action is universal in the sense that any Goldstino model should be related to the AV action by a nonlinear field redefinition. Various Goldstino models can be interesting in their own right,³ in particular, those models which are realized in terms of constrained superfields. Over the years, there have appeared a number of such superfield actions with spontaneously broken supersymmetry [11, 12, 13, 14].

Recently there has been renewed interest in Goldstino couplings inspired by the work of Komargodski and Seiberg [15]. They put forward the Goldstino model that had actually appeared in the literature twenty years earlier [14]. The novelty of the Komargodski-Seiberg (KS) approach is that they related the Goldstino dynamics to the superconformal anomaly multiplet X corresponding to the Ferrara-Zumino supercurrent [16]. Under the renormalization group flow, the multiplet of anomalies X defined in the UV turns out to flow in the IR to a chiral superfield X_{NL} (obeying the constraint $X_{NL}^2 = 0$, of the type first introduced by Roček [11]) which contains the Goldstino as a component field. For a $\mathcal{N} = 2$ generalization of the KS formalism, see [17].

The action derived in [14, 15] has a particularly simple form both in superspace and when reduced to components. However, its direct relation to the AV action and thus the structure of its nonlinearly realized supersymmetry have not yet been studied.

In [18] it was shown, using the general method of [4, 5], that the Goldstino action introduced by Samuel and Wess [13] can be derived from the constrained superfield formalism of Komargodski and Seiberg. The former model is known to be equivalent to the AV theory [13].

What we provide in this paper is a direct relation between the AV action and that of Komargodski-Seiberg. Unlike [18], we do not make use of the techniques developed in [4, 5]. Instead we follow the approach pursued in [10] which can also be applied to study the fermionic sector of supersymmetric Euler-Heisenberg-type actions. We use the two-component notation and conventions adopted in [2, 19].

³ For instance, the fermionic sector of the $\mathcal{N} = 1$ supersymmetric Born-Infeld action [7, 8] is a new Goldstino model. It has been shown to be related to the AV action by a nonlinear field redefinition [9, 10].

2 Setup and results

The AV action [1] is

$$S_{\text{AV}}[\lambda, \bar{\lambda}] = \frac{1}{2\kappa^2} \int d^4x \left(1 - \det \Xi \right), \quad (1)$$

where κ denotes the dimensionful coupling constant and

$$\Xi_a{}^b = \delta_a{}^b + \kappa^2 (v + \bar{v})_a{}^b, \quad v_a{}^b = i\lambda\sigma^b\partial_a\bar{\lambda}, \quad \bar{v}_a{}^b = -i\partial_a\lambda\sigma^b\bar{\lambda}. \quad (2)$$

By construction, S_{AV} is invariant under the nonlinear supersymmetry transformations

$$\delta_\xi \lambda_\alpha = \frac{1}{\kappa} \xi_\alpha - i\kappa (\lambda\sigma^a\bar{\xi} - \xi\sigma^a\bar{\lambda}) \partial_a \lambda_\alpha. \quad (3)$$

Expanding out the determinant in (1) and denoting the trace of a matrix $M = (M_a{}^b)$ with Lorentz indices as $\langle M \rangle = \text{tr}(M) = M_a{}^a$ yields

$$\begin{aligned} S_{\text{AV}}[\lambda, \bar{\lambda}] = & -\frac{1}{2} \int d^4x \left(\langle v + \bar{v} \rangle + 2\kappa^2 \left(\langle v \rangle \langle \bar{v} \rangle - \langle v\bar{v} \rangle \right) \right. \\ & \left. + \kappa^4 \left(\langle v^2\bar{v} \rangle - \langle v \rangle \langle v\bar{v} \rangle - \frac{1}{2} \langle v^2 \rangle \langle \bar{v} \rangle + \frac{1}{2} \langle v \rangle^2 \langle \bar{v} \rangle + \text{c.c.} \right) \right). \end{aligned} \quad (4)$$

As demonstrated in [10], the 8th-order terms vanish.

The Goldstino action constructed in [14, 15] is

$$S_{\text{KS}}[\psi, \bar{\psi}] = -\frac{1}{2} \int d^4x \left(\langle u + \bar{u} \rangle + \frac{1}{2f^2} \partial^a \bar{\psi}^2 \partial_a \psi^2 + \frac{1}{8f^6} \psi^2 \bar{\psi}^2 \partial^2 \psi^2 \partial^2 \bar{\psi}^2 \right), \quad (5)$$

where we defined $u_a{}^b = i\psi\sigma^b\partial_a\bar{\psi}$ and its complex conjugate. In the following section, we find that the constant f is related to κ via $2f^2 = \kappa^{-2}$.

Below, we find that the nonlinear field redefinition which maps the action (1) to the action (5), i.e. $S_{\text{AV}}[\lambda_\alpha(\psi, \bar{\psi}), \lambda_\alpha(\psi, \bar{\psi})] = S_{\text{KS}}[\psi, \bar{\psi}]$, can be chosen to be

$$\begin{aligned} \lambda_\alpha(\psi, \bar{\psi}) = & \psi_\alpha - i\frac{\kappa^2}{2} (\sigma^a \bar{\psi})_\alpha (\partial_a \psi^2) - \frac{\kappa^4}{2} \psi_\alpha \left(\langle u\bar{u} \rangle - 2\langle u \rangle \langle \bar{u} \rangle + \frac{1}{2} \langle \bar{u}^2 \rangle - \frac{1}{2} \partial^a \psi^2 \partial_a \bar{\psi}^2 \right. \\ & \left. + \frac{1}{4} \bar{\psi}^2 \square \psi^2 \right) + \kappa^6 \psi_\alpha \left(\langle u\bar{u}^2 \rangle + \frac{3}{2} \langle u\bar{u} \rangle \langle \bar{u} \rangle + \frac{3}{4} \langle u \rangle \langle \bar{u}^2 \rangle \right). \end{aligned} \quad (6)$$

The inverse field redefinition is

$$\begin{aligned} \psi_\alpha(\lambda, \bar{\lambda}) = & \lambda_\alpha + i\frac{\kappa^2}{2} (\sigma^a \bar{\lambda})_\alpha (\partial_a \lambda^2) \left(1 + \kappa^2 \langle \bar{v} \rangle \right) \\ & + \frac{\kappa^4}{2} \lambda_\alpha \left(\langle v\bar{v} \rangle - \frac{1}{2} \langle \bar{v}^2 \rangle - \langle \bar{v} \rangle^2 + \frac{1}{2} \partial^a \lambda^2 \partial_a \bar{\lambda}^2 + \frac{3}{4} \bar{\lambda}^2 \square \lambda^2 \right) \\ & - \kappa^6 \lambda_\alpha \left(\langle v\bar{v}^2 \rangle + \frac{1}{2} \langle v\bar{v} \rangle \langle \bar{v} \rangle - \frac{1}{2} \langle v \rangle \langle \bar{v}^2 \rangle - \frac{1}{4} \langle v \rangle \langle \bar{v} \rangle^2 + \frac{3}{4} \langle \bar{v} \rangle \partial^a \lambda^2 \partial_a \bar{\lambda}^2 \right). \end{aligned} \quad (7)$$

3 Deriving the nonlinear field redefinition

In this section, we sketch the derivation of (6). A more detailed presentation of our method will be given in a separate publication.

Our goal is to find a nonlinear field redefinition $\lambda_\alpha \rightarrow \lambda_\alpha(\psi, \bar{\psi}) = \psi_\alpha + O(\kappa^2)$ that satisfies

$$S_{\text{AV}}[\lambda(\psi, \bar{\psi}), \bar{\lambda}(\psi, \bar{\psi})] \equiv \tilde{S}_{\text{AV}}[\psi, \bar{\psi}] = S_{\text{KS}}[\psi, \bar{\psi}] . \quad (8)$$

Since both actions $S_{\text{AV}}[\lambda, \bar{\lambda}]$ and $S_{\text{KS}}[\psi, \bar{\psi}]$ are invariant under R -symmetry, the nonlinear transformation we are looking for must be covariant under R -symmetry. The most general field transformation of this type is

$$\begin{aligned} \lambda_\alpha(\psi, \bar{\psi}) = & \psi_\alpha + \kappa^2 \psi_\alpha \langle \alpha_1 u + \alpha_2 \bar{u} \rangle + i\kappa^2 (\sigma^a \bar{\psi})_\alpha (\partial_a \psi^2) \left(\alpha_3 + \kappa^2 \langle \beta_7 u + \beta_8 \bar{u} \rangle \right) \\ & + \kappa^4 \psi_\alpha \left(\beta_1 \langle u \bar{u} \rangle + \beta_2 \langle u \rangle \langle \bar{u} \rangle + \beta_3 \langle \bar{u}^2 \rangle + \beta_4 \langle \bar{u} \rangle^2 + \beta_5 \partial^a \psi^2 \partial_a \bar{\psi}^2 + \beta_6 \bar{\psi}^2 \square \psi^2 \right) \\ & + \kappa^6 \psi_\alpha \left(\gamma_1 \langle u \bar{u}^2 \rangle + \gamma_2 \langle u \bar{u} \rangle \langle \bar{u} \rangle + \gamma_3 \langle u \rangle \langle \bar{u}^2 \rangle + \gamma_4 \langle u \rangle \langle \bar{u} \rangle^2 + \gamma_5 \langle \bar{u} \rangle \partial^a \psi^2 \partial_a \bar{\psi}^2 \right) . \end{aligned} \quad (9)$$

This is equivalent to the field redefinition used in [10] up to some 7-fermion identities.

The general field redefinition at $O(\kappa^2)$ acts on the AV action to give

$$\begin{aligned} \tilde{S}_{\text{AV}} = & - \int d^4x \left\{ \frac{1}{2} \langle u + \bar{u} \rangle + \kappa^2 \left(\frac{1}{2} (\langle u \rangle^2 - \langle u^2 \rangle + \text{c.c.}) + ((\alpha_1 + \alpha_3) \langle u \rangle^2 + \text{c.c.}) \right. \right. \\ & \left. \left. - (\alpha_3 \langle u^2 \rangle + \text{c.c.}) + 2\text{Re}(\alpha_2) \langle u \rangle \langle \bar{u} \rangle - \text{Re}(\alpha_3) \partial^a \psi^2 \partial_a \bar{\psi}^2 \right) + O(\kappa^4) \right\} , \end{aligned} \quad (10)$$

where we have rewritten all terms in the minimal basis

$$\langle u^2 \rangle, \quad \langle \bar{u}^2 \rangle, \quad \langle u \rangle \langle \bar{u} \rangle, \quad \langle u \rangle^2, \quad \langle \bar{u} \rangle^2, \quad \partial^a \psi^2 \partial_a \bar{\psi}^2 . \quad (11)$$

Obviously, if we are to match S_{KS} to this order we need

$$\alpha_1 = 0, \quad \text{Re}(\alpha_2) = 0, \quad \alpha_3 = -\frac{1}{2}, \quad 2f^2 = \kappa^{-2} . \quad (12)$$

The imaginary part of α_2 , which we will denote as α_2^i , is not fixed at this order.

The effect of (9) with (12) on the AV action at $O(\kappa^4)$ can be similarly analysed. If we split all coefficients into their real and imaginary parts, $\beta_j = \beta_j^r + i\beta_j^i$, then the restrictions on the β_j can be written as

$$\begin{aligned} \beta_1^r &= 4\beta_6^r + 2\beta_8^r, \quad \beta_1^i = 2\alpha_2^i + 4\beta_6^i - 2\beta_8^i, \quad \beta_3^r = -\frac{1}{2}(1 + 4\beta_6^r), \quad \beta_3^i = -2(\alpha_2^i + \beta_6^i), \\ \beta_2^r &= \frac{3}{2} - \beta_4^r + 4\beta_6^r - \beta_7^r - \beta_8^r, \quad \beta_2^i = -\frac{1}{2}\alpha_2^i + \beta_4^i - \beta_7^i + \beta_8^i, \\ \beta_5^r &= \frac{1}{2} + 2\beta_6^r + \beta_8^r, \quad \beta_5^i = \alpha_2^i + 2\beta_6^i - \beta_8^i, \quad \beta_6^r = -\frac{1}{8}(1 + (\alpha_2^i)^2) . \end{aligned} \quad (13)$$

The seven real parameters $\beta_4^r, \beta_4^i, \beta_6^i, \beta_7^r, \beta_7^i, \beta_8^r$ and β_8^i are not fixed at this order.

A similar analysis is performed at $O(\kappa^6)$ and we find that to match \tilde{S}_{AV} to S_{KS} we need

$$\begin{aligned}\gamma_1 &= 1, & \gamma_2^r &= \frac{3}{2} - 2\alpha_2^i\left(\frac{1}{4}\alpha_2^i + 2\beta_6^i - \beta_8^i\right) + 2(\beta_7^r + \beta_8^r + \gamma_5^r), \\ \gamma_3^r &= \frac{3}{4} - \alpha_2^i\left(\frac{1}{4}\alpha_2^i + 2\beta_6^i + \beta_7^i - \beta_8^i\right) + 2\beta_4^r + 3\beta_8^r, \\ \gamma_3^i &= -\alpha_2^i\left(\frac{1}{4}(\alpha_2^i)^2 + \frac{9}{4} - \beta_7^r - \beta_8^r\right) - 2\beta_4^i - 6\beta_6^i - \beta_8^i, \\ \gamma_4^r &= -\alpha_2^i\left(\alpha_2^i + \frac{3}{2}\beta_4^i + 2\beta_6^i + \frac{1}{2}\beta_7^i + \frac{3}{2}\beta_8^i\right) + \frac{1}{2}(\beta_4^r - \beta_7^r + \beta_8^r).\end{aligned}\tag{14}$$

The free parameters at this order are $\gamma_2^i, \gamma_4^i, \gamma_5^i$ and γ_5^r , the first three of which have completely dropped out the calculation.

It can be shown that all the free parameters can be accounted for by the symmetries of either one of the two actions. In particular γ_2^i, γ_4^i and γ_5^i correspond to single term trivial symmetries of any Goldstino action.

From the above results we see that out of the original 32 real parameters in the nonlinear field redefinition, 12 remain unfixed by the requirement that $\tilde{S}_{AV} = S_{KS}$. Since these freedoms may be recovered by a symmetry transformation of either action, we may simply set all free parameters to zero and get the field redefinition (6).

Some results of this section were obtained with computer assistance [20]. The core of the computer program is the generation of a canonical form for expressions involving spinors, which is necessary for comparing expressions. All Fierz-type identities were automatically satisfied by choosing a representation for the Pauli matrices and defining a definite ordering for spinors and their derivatives. Total derivatives, where relevant, were removed from expressions by generating a set of replacement rules that performed the appropriate integration by parts to yield a unique form for the expression. Further details of the algorithm will be given in a separate publication.

4 Concluding comments

It has been pointed out, e.g. [14, 18], that S_{KS} does not have definite transformation properties under the supersymmetry transformation (3). But now that we have an explicit mapping from S_{AV} to S_{KS} we can use it to find the supersymmetry transformation under

which S_{KS} is invariant. We get

$$\begin{aligned} \delta_\xi \psi_\alpha &= \delta_\xi \psi_\alpha(\lambda, \bar{\lambda}) = \delta_\xi \lambda^\beta \cdot \frac{\delta}{\delta \lambda^\beta} \psi_\alpha(\lambda, \bar{\lambda}) + \delta_\xi \bar{\lambda}_{\dot{\beta}} \cdot \frac{\delta}{\delta \bar{\lambda}_{\dot{\beta}}} \psi_\alpha(\lambda, \bar{\lambda}) \Big|_{\lambda=\lambda(\psi, \bar{\psi})} \\ &= \frac{1}{\kappa} \xi_\alpha - i\kappa \left((\psi \sigma^a \bar{\xi} - \xi \sigma^a \bar{\psi}) \partial_a \psi_\alpha - (\sigma^a \bar{\psi})_\alpha \partial_a (\xi \psi) - \frac{1}{2} (\sigma^a \bar{\xi})_\alpha \partial_a \psi^2 \right) + O(\kappa^3) . \end{aligned} \quad (15)$$

Finally, we would like to comment on the field redefinition found by Zheltukhin [21, 22]. In these papers, written in the four-component spinor notation, the required field redefinition was sought in the form⁴ $\boldsymbol{\psi}(\boldsymbol{\lambda}) = \boldsymbol{\lambda} + \kappa^2 \boldsymbol{\chi}(\boldsymbol{\lambda}) + O(\kappa^4)$. By requiring that $S_{\text{KS}}[\boldsymbol{\psi}(\boldsymbol{\lambda})] = S_{\text{AV}}[\boldsymbol{\lambda}]$ a solution was found for $\boldsymbol{\chi}$. The key step in the $O(\kappa^2)$ calculation reported in [21] is the factorisation

$$(\partial^m \bar{\boldsymbol{\lambda}})(\gamma_m \boldsymbol{\chi} + \boldsymbol{\zeta}_m(\boldsymbol{\lambda})) = 0 , \quad (16)$$

where we have introduced

$$\boldsymbol{\zeta}_m(\boldsymbol{\lambda}) = \frac{i}{2} (\boldsymbol{\lambda}(\bar{\boldsymbol{\lambda}}_{,m} \boldsymbol{\lambda}) + \gamma_5 \boldsymbol{\lambda}(\bar{\boldsymbol{\lambda}}_{,m} \gamma_5 \boldsymbol{\lambda})) - \frac{i}{4} (\gamma_m \boldsymbol{\lambda}(\bar{\boldsymbol{\lambda}}_{,n} \gamma^n \boldsymbol{\lambda}) - \gamma^n \boldsymbol{\lambda}(\bar{\boldsymbol{\lambda}}_{,n} \gamma_m \boldsymbol{\lambda})) \quad (17)$$

and denoted by $\bar{\boldsymbol{\lambda}}_{,m}$ the derivative of $\bar{\boldsymbol{\lambda}}$ with respect to x^m . In [21, 22], it was then inferred from (16) that

$$\gamma_m \boldsymbol{\chi} + \boldsymbol{\zeta}_m(\boldsymbol{\lambda}) = 0 . \quad (18)$$

Unfortunately, the 16 equations (18) for the 4 components of $\boldsymbol{\chi}$ are inconsistent. This can be seen by taking a time- or space-like vector p^m and contracting both sides of (18) with $(p^n \gamma_n)^{-1} p^m = -p^{-2} (p^n \gamma_n) p^m$. Then, the first term in the relation obtained will be p -independent, while the second remains p -dependent.

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⁴We use bold-face Greek letters for four-component spinors.

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